

$$1 \text{ a } \mathbf{a} - \mathbf{b} = (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \\ = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

$$\text{b } 3\mathbf{b} - 2\mathbf{a} + \mathbf{c} = 3(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) - 2(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + (-\mathbf{i} + \mathbf{k}) \\ = 6\mathbf{i} - 3\mathbf{j} + 9\mathbf{k} - 2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} - \mathbf{i} + \mathbf{k} \\ = 3\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$$

$$\text{c } |\mathbf{b}| = \sqrt{2^2 + (-1)^2 + 3^2} \\ = \sqrt{4 + 1 + 9} \\ = \sqrt{14}$$

$$\text{d } |\mathbf{b} + \mathbf{c}| = |(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + (-\mathbf{i} + \mathbf{k})| \\ = |\mathbf{i} - \mathbf{j} + 4\mathbf{k}| \\ = \sqrt{1^2 + (-1)^2 + 4^2} \\ = \sqrt{18} = 3\sqrt{2}$$

$$\text{e } 3(\mathbf{a} - \mathbf{b}) + 2\mathbf{c} = 3((\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 3\mathbf{k})) + 2(-\mathbf{i} + \mathbf{k}) \\ = 3(-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) - 2\mathbf{i} + 2\mathbf{k} \\ = -3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k} - 2\mathbf{i} + 2\mathbf{k} \\ = -5\mathbf{i} + 6\mathbf{j} - \mathbf{k}$$

$$2 \text{ a } \vec{OB} = \vec{OA} + \vec{OC} \\ = 2\mathbf{j} + 2\mathbf{k}$$

$$\text{b } \vec{OE} = \vec{OA} + \vec{OD} \\ = \mathbf{i} + 2\mathbf{j}$$

$$\text{c } \vec{OG} = \vec{OC} + \vec{OD} \\ = \mathbf{i} + 2\mathbf{k}$$

$$\text{d } \vec{OF} = \vec{OA} + \vec{OC} + \vec{OD} \\ = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\text{e } \vec{ED} = -\vec{OA} \\ = -2\mathbf{j}$$

$$\text{f } \vec{EG} = -\vec{OA} + \vec{OC} \\ = -2\mathbf{j} + 2\mathbf{k}$$

$$\text{g } \vec{CE} = -\vec{OC} + \vec{OA} + \vec{OD} \\ = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\text{h } \vec{BD} = -\vec{OC} - \vec{OA} + \vec{OD} \\ = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$$

$$3 \text{ a i } |\mathbf{a}| = \sqrt{3^2 + 1^2 + 1^2} \\ = \sqrt{11} \\ \hat{\mathbf{a}} = \frac{1}{\sqrt{11}} (3\mathbf{i} + \mathbf{j} - \mathbf{k}) \\ = \frac{3}{\sqrt{11}}\mathbf{i} + \frac{1}{\sqrt{11}}\mathbf{j} - \frac{1}{\sqrt{11}}\mathbf{k}$$

$$\text{ii } -2\hat{\mathbf{a}} = -\frac{6}{\sqrt{11}}\mathbf{i} - \frac{2}{\sqrt{11}}\mathbf{j} + \frac{2}{\sqrt{11}}\mathbf{k}$$

$$\text{b } 5\hat{\mathbf{a}} = \frac{15}{\sqrt{11}}\mathbf{i} + \frac{5}{\sqrt{11}}\mathbf{j} - \frac{5}{\sqrt{11}}\mathbf{k}$$

$$4 \quad |\mathbf{a}| = \sqrt{1^2 + 1^2 + 5^2}$$

$$= \sqrt{27} = 3\sqrt{3}$$

$$|\mathbf{b}| = \sqrt{2^2 + 1^2 + 3^2}$$

$$= \sqrt{14}$$

$$\mathbf{c} = \frac{|\mathbf{a}|}{|\mathbf{b}|}\mathbf{a}$$

$$= \frac{\sqrt{14}}{3\sqrt{3}}(\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

$$= \frac{\sqrt{42}}{9}(\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

$$5 \text{ a } \vec{PQ} = \mathbf{i} - 3\mathbf{j}$$

$$\text{b } |\vec{PQ}| = \sqrt{1^2 + 3^2 + 0^2}$$

$$= \sqrt{10}$$

$$\text{c } \vec{OM} = \vec{OP} + \vec{PM}$$

$$= \vec{OP} + \frac{1}{2}\vec{PQ}$$

$$= \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \frac{1}{2}\mathbf{i} - \frac{3}{2}\mathbf{j}$$

$$= \frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$$

$$6 \text{ a } \vec{OE} = \vec{OA} + \vec{AE}$$

$$= \mathbf{i} + 3\mathbf{j}$$

$$\vec{OM} = \frac{1}{3}\vec{OE}$$

$$= \frac{1}{3}\mathbf{i} + \mathbf{j}$$

$$\vec{BF} = \vec{OD}$$

$$= \mathbf{i}$$

$$\vec{BN} = \frac{1}{2}\vec{BF}$$

$$= \frac{1}{2}\mathbf{i}$$

$$\vec{ON} = \vec{OC} + \vec{CB} + \vec{BN}$$

$$= \frac{1}{2}\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

$$\vec{MN} = \vec{ON} - \vec{OM}$$

$$= \frac{1}{2}\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} - \left(\frac{1}{3}\mathbf{i} + \mathbf{j}\right)$$

$$= \frac{1}{6}\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\text{b } |\vec{MN}| = \sqrt{\left(\frac{1}{6}\right)^2 + 2^2 + 2^2}$$

$$\begin{aligned} &= 4\sqrt{\frac{1 + 144 + 144}{36}} \\ &= \sqrt{\frac{289}{36}} \\ &= \frac{17}{6} \end{aligned}$$